Revista Publicando, 5 No 16. (1). 2018, 855-860. ISSN 1390-9304

A Simple Method to Calculate Time Delay Due to the Sun Gravitational Field

Ehsan Bashiri[1]

1 NIOPDC, Ilam, Iran, ebsh.ait@gmail.com

Abstract—Although some experimental and theoretical methods have been discovered to measure time delay of a beam of light passing close to the Sun caused by its gravitational field, one interesting subject is still remained that could be under consideration; this is difference between times derived from Schwarzschild metric and what is shall be recorded if no gravity affects. This difference is dealt with as a hypothetical metric. It is obvious that in reality space is not flat and gravitational field impacts spacetime; however, assuming and considering a flat spacetime and analyzing its difference from Schwarzschild metric helps to understand how much time is delayed due to General Relativistic effects of objects of the universe. What is calculated in this paper is the amount in existence of the Sun alone. Effects of entire the universe could be subject of future studies.

Keywords—time delay,spacetime, Schwarzschild metrics, General Relativity

Received 01/06/2018
Approved 05/07/2018

I. INTRODUCTION

SPACETIME has been one of the crucial concerns of contemporary scientists and researchers, also one of the valuable tools for developing other branches of Physics that emerged subsequently during the last century. Most of the popularity of spacetime owes Special and General Relativity which coincided to the landmark of the Physics at the beginning of 20th Century.

Special Relativity formulates structure of Minkowski spacetime in a flat vacuum space distant from any massive object with zero cosmological constant and energy-momentum tensor [1]. While in General Relativity, Schwarzschild metric describes existence of gravitational field and its effect on the structure of space around any spherical rotating or non-rotating massive object, again in a vacuum space [2].

It is known to all how gravitational field of the Sun affects and deflects path of a beam of light passing in vicinity of a mass according to the Schwarzschild metric. Moreover, curvature induced due to the gravity causes a delay in time which seems to reduce speed of light examined by Shapiro [3].

What human can observe is that of exists in the real world due to effects of gravitational fields which surrounds him. This means that human could observe different things, or at least at different times sooner or later, if gravitational field subside or become denser. This paper tries to measure quantity of this time delay, and verify whether there exists any formalized relationship or not.

II. CALCULATING TIME DELAY

Looking at the figure below, two beams of light are considered to be emitted from an infinitely distant star passing near the Sun:

i is a real beam of light corresponding to Schwarzschild Metric [4],

$$d\tau_i^{\,2} = (1-\frac{2m}{r_i})dt_i^{\,2} - (1-\frac{2m}{r_i})^{-1}dr_i^{\,2} - r_i^{\,2}(d\theta^2 + \sin^2(\theta)d\phi^2) \qquad (1)$$

j is an imaginary beam of light, geodesic-apart from any massive object sweeping a straight line on a flat surface,

Received 01/06/2018
Approved 05/07/2018

Revista Publicando, 5 No 16. (1). 2018, 855-860. ISSN 1390-9304

$$d\tau_j{}^2 = dt_j{}^2 - dr_j{}^2 - r_j{}^2(d\theta^2 + \sin^2(\theta)d\phi^2) \qquad (2)$$

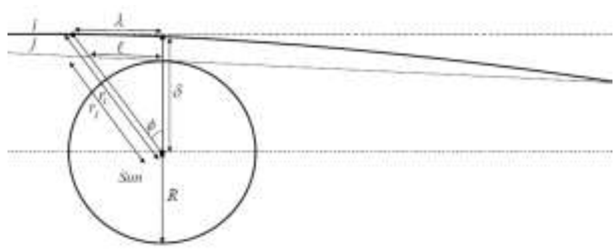

Fig.1 Schematic of two beams of light passing near the Sun

Conditions: $r\phi$ plane and $\theta = \dfrac{\pi}{2}$

From the figure above, using Pythagorean Theorem, following relationships could be obtained

$$\begin{cases} r_i{}^2 = \delta^2 + \lambda^2 \\ r_j{}^2 = R^2 + \ell^2 \end{cases} \& \frac{r_i}{r_j} = \frac{\lambda}{\ell} = \frac{\delta}{R}$$
$$\cos\phi = \frac{\delta}{r_i} = \frac{R}{r_j} \Rightarrow d\phi = \frac{\delta}{\lambda r_i}dr_i = \frac{R}{\ell r_j}dr_j \qquad (3)$$

It must be denoted that Pythagorean relations are used for simplification. There is no doubt that the spacetime is curved due to gravity and therefore, readers should not need to be confused!

Inserting these relationships into the flat metric and rewriting equation (2) in terms of quantities of *i* will give

$$\begin{cases} d\tau_i{}^2 = (1-\dfrac{2m}{r_i})dt_i^2 - (1-\dfrac{2m}{r_i})^{-1}dr_i^2 - r_i{}^2 d\phi^2 \\ d\tau_j{}^2 = dt_j{}^2 - (\dfrac{R}{\delta})^2 dr_i^2 - (\dfrac{R}{\delta})^2 r_i{}^2 d\phi^2 \end{cases} \qquad (4)$$

there's no need to prove that [5],

$$(\frac{R}{\delta})^2 = (1-\frac{2m}{r_i}) \qquad (5)$$

this changes equation (4) to

Received 01/06/2018
Approved 05/07/2018

$$\begin{cases} d\tau_i^{\,2} = (1-\dfrac{2m}{r_i})dt_i^{\,2} - (1-\dfrac{2m}{r_i})^{-1}dr_i^{\,2} - r_i^{\,2}d\phi^2 \\ d\tau_j^{\,2} = dt_j^{\,2} - (1-\dfrac{2m}{r_i})dr_i^{\,2} - (1-\dfrac{2m}{r_i})r_i^{\,2}d\phi^2 \end{cases} \qquad (6)$$

Although it is known that according to the General Relativity velocity of light is not mandatorily equal to *c* and so, proper time of light is not zero, here only Special relativistic effects is considered, i.e. $d\tau_i = d\tau_j = 0$; hence

$$\begin{cases} dt_i^{\,2} = (1-\dfrac{2m}{r_i})^{-2}dr_i^{\,2} + (1-\dfrac{2m}{r_i})^{-1}r_i^{\,2}d\phi^2 \\ dt_j^{\,2} = (1-\dfrac{2m}{r_i})dr_i^{\,2} + (1-\dfrac{2m}{r_i})r_i^{\,2}d\phi^2 \end{cases}$$

inserting (3) quantities and rewriting above relationships based on result of division of dt_i on dt_j gives rise to

$$(\frac{dt_i}{dt_j})^2 = \frac{(1-\dfrac{2m}{r_i})^{-2} + (1-\dfrac{2m}{r_i})^{-1}(\dfrac{\delta}{\lambda})^2}{(1-\dfrac{2m}{r_i}) + (\dfrac{R}{\lambda})^2} \qquad (7)$$

Based on the overall deviation of light passing near the Sun resulting from the Einstein prediction [2], it is known that

$$\phi = \frac{4M}{R} \qquad (8)$$

Assuming that the under-experiment beam of the light passes tangent to the Sun, i.e. $r_i \approx r_j = R$:

$$\sin\phi \approx \phi \Rightarrow \frac{\ell}{R} = \frac{4M}{R} \;\&\; \lambda = (\frac{\delta}{R})\ell$$

considering $k = \dfrac{M}{R}$, and using an important principle in mathematics [4]

$$(1+\varepsilon)^p = 1 + p\varepsilon \qquad (9)$$

(7) simplifies to

$$(\frac{dt_i}{dt_j})^2 = \frac{1+4k+20k^2+96k^3}{1+16k^2}$$

ignoring 2nd and 3rd powers of *k* that are puny,

Received 01/06/2018
Approved 05/07/2018

$$(\frac{dt_i}{dt_j})^2 \approx 1+4k \approx (1-\frac{2M}{R})^{-2}$$

in other words;

$$\frac{dt_i}{dt_j} = (1-\frac{2m}{r_i})^{-1} \qquad (10)$$

comparing with (5), (10) would be simplified to

$$\left.\frac{dt_i}{dt_j}\right|_{t=0}^{t=\infty} = (1-\frac{2m}{r_i})^{-1} \Big|_{r=\infty}^{r=R} \approx (\frac{\delta}{R})^2 \qquad (11)$$

(11) implies that

$$t_i - t_j = ((\frac{\delta}{R})^2 - 1)t_j \pm Cons.$$

$$= \frac{2m}{r_i} t_j \pm Cons. \qquad (12)$$

δ and R are shown in Fig.1, t_j is time of reaching sunlight to earth (500 *s*), and $Cons.$ is constant value of integral.

Inserting astronomical constants *G, M_s, c, R_s*, and solving for $Cons. = 0$

$$t_i - t_j \approx 2.1ms$$

In other words, time delay measured due to the Sun gravitational field_ neglecting other gravities induced by all cosmological masses is calculated almost 2.1 *m* seconds.

III. DISCUSSIONS &CONCLUSION

Entire this paper is based upon a hypothesis namely difference of two Schwarzschild and flat metrics as a new metric. It is clear that this metric does not exist in reality; however, it was created in order to answer a question: How much time is delayed due to existence of gravitational field of massive objects of the universe? Or: how many seconds, minutes or hours human are back from collecting information from other galaxies? This is a really controversial question and its answer is not exact; but effect of the Sun could be assessed.

Time required for a beam of light emitted from an outer source to reach Earth could be easily calculated by knowing the distance; however, what is more important is to find out how long it will take to reach an earthy observer if gravity does not exist. Time delay which is calculated in this paper is due to existence of the Sun alone. What calculated in this paper was almost to be 2.1

Received 01/06/2018
Approved 05/07/2018

m seconds. This is 10 times greater than what Shapiro measured experimentally [6]. The reason of this inaccuracy is that in this paper the unit vectors pointing from observer to the source and also to the Sun have not been considered. Along with other limitations assumed like neglecting longitudinal and latitudinal dimensions, dragging spacetime or *Kerr Solution*, and in other words, neglecting all General Relativistic effects surrounding a rotating, spherically massive object.

REFERENCES

[1] A. Einstein, *Relativity; The meaning of Relativity*, Khwarizmi Press, Iran, 1984.

[2] A. Einstein, *General Relativity*, Khwarizmi Press, Iran, 1984.

[3] C. M. Will, *Was Einstein Right?*, *Putting General Relativity to the Test*, Ghoghnus Publications, Iran, 1995.

[4] L. D. Landau, E. M. Lifshitz, *A Shorter Course of Theoretical Physics*, *Volume 1, Mechanics and Electrodynamics*, *First edition*, Razi Press, Iran, 1972.

[5] Christian, Magnan, *Deviation of Light Near the Sun in General Relativity*, (France, 2007), online at: http://www.lacosmo.com/DeflectionOfLight/index.html.

[6] Irwin I. Shapiro, 1964*Fourth Test of General Relativity*, Physical Review Letters 13 (26): 789–791

Received 01/06/2018
Approved 05/07/2018